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MODELLING OF THE GROUNDWATER FLOW IN FRACTURED ROCK — A NEW APPROACH. *

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Abstract. A new approach to the modelling of the groundwater flow in fractured rock is presented in the paper. The empirical knowledge of the hydrogeologists is summarized first. There are three types of objects important for the groundwater flow — small fractures, which can be replaced by blocks of porous media, large deterministic fractures and lines of intersection of the large fractures. These objects are by their nature 3D, resp. 2D, resp. 1D. The rest of the paper describes how to set up a numerical model representing all three types of objects. We use existing models based on the Mixed-Hybrid FEM and we connect them by equations representing the mass exchange between various types of elements. Our model uses two types of the connection of the elements, so-called *compatible* and *incompatible* type.

Key words. fractured rock, groundwater flow, finite element method, mixed-hybrid FEM

AMS subject classifications. 65N30, 76S05

1. Introduction and motivation. Numerical modelling of the hydraulical, geochemical and transport processes in the fractured rock attracts the attention of many scientists for more than forty years. The first numerical models of such processes were created in late 60's of the 20-th century. According to [7], there existed more than thirty software packages claimed to solve problem of the fluid flow in fractured rock in 1994.

Despite these facts, there is a lot of open and unresloved problems in this field of research. The reason for such situation lies in the nature of the problem. Lack of input data, their uncertainity and often low accuracy, high computational costs are the main difficutlies we encounter when we try to simulate processes in the fractured rock. Avoiding these difficulties is usally possible only at a price of simplifications of the problem.

Our research is motivated by the need of finding the most suitable locality for a permanent deep repository of the radioactive waste. There are two nuclear power plants in the Czech Republic, a construction of the repository is planned in the 30's of the 21-th century. Neverthless, the process of selection of the most suitable locality already began, as well as some other preliminary projects. Two of them are projects GAČR 102/04/P019 and MŽP VaV 660/2/03 focused on improving and testing existing numerical models and development of the new models. In this paper, we will show one of the results of these projects, a new approach for numerical models of groundwater flow which could be used for simulations of the processes in the large neighbourhood of the repository.

2. Principal ideas of the model of fluid flow in fractured rock environment. The radioactive waste repository will be situated in the copmact crystalinic rock massif. Of course, a good and reliable numerical model of the fluid flow and

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transport in such massif has to reflect its specific properties. The hydrogeologc research brought following empirical knowledge about the rock environment there and groundwater flow in them.

- The rock matrix can be considered hydraulicaly impermeable.
- Even the most compact massifs are disrupted by numerous fractures.
- Most of these fractures are relatively small ones, with the characteristic length less than one meter.
- The groundwater flow in the small fractures is extremely slow.
- On the other hand, these small fractures have significant storativity capacity and play an important role in the transport processes.
- It is barly possible to obtain exact data of all the small fractures. They must be treated only in statistical way.
- The most of the liquid is conducted by relatively small number of large fractures. The spatial position of these fractures is usually known or detectable by the field measurements.
- The fastest flux of groundwater is observed on intersections of the large fractures. These lines of the intersection behaves like a "pipelines" in the compact rock massifs.

These facts lead us to conclusion that there are three different types of objects involved in conduction of the groundwater through the compact rock: small fractures, large fractures and intersections of the large fractures. Now let us examine these objects form the point of view of the numerical modelling.

2.1. The small fractures. As said before, in most cases there is large number of the small fractures in the massif. However we usually know only the data of statistical kind (such as distribution of poles) about that fractures. There are two possible approaches to the modelling of the flow in such environment: *the stochastic discrete fracture networks* or *the homogenization and replacement with porous media.*

The first one is more suitable for small problems (spatial dimension of the domain up to tens of meter) but for the large problems (such are simulations of the massif with the repository) we encounter serious problems (mainly of the comutational nature) with using that approach. For more information about this approach see for example [5].

On the other hand, the second approach is much better applicable for large problems. Fractured rock dissrupted only by small fractures can be relatively well homogenized and replaced by a porous media with equivalent hydraulical properties. The methods of the homogenization and setting the hydraulical parameters of the replacing porous media can be found for example in [3] or [9] or [4]

2.2. The large fractures. The situation here is almost an opposite of the previous one. The large fractures are relatively well known and not numerous but causing strong heterogenity of the environment. The methods of homogenization leads to the serious errors and inaccuracies in this case. However, the *discrete fracture networks* approach (not stochastic) works well even for large problems if we consider only the large fractures.

The DFN approach usually represents the fractures as two-dimensional objects (circular discs, poygons, etc...) placed in the 3D space. The transversal dimension of the fractures is at least hunderd times smaller than the other two dimensions, so the representation as 2D objects causes no significant loss of accuracy of the model. The transversal dimension of the fracture effects the values of permeability tensor as shown in [3] or [2]

2.3. The intersections of the large fractures. This case is similar to the previous one. The objects of this type is relatively rare in the rock massif, but significant for the fluid flow. The velocity of the flow on the intersection of fractures can be higher in order of magnitude than the velocity in the fractures. Fortunately, the velocity is still low enough for holding the assumption of the potential flow governed by the Dary's law. For the same reasons as in the previous case we can treat the intersections as one-dimensional objects placed in 3D space.

As a result of the previous paragraphs, we can say that the model of groundwater flow in the compact rock massifs should incorporate 3D porous blocks, 2D fractures and 1D lines. So we have three different domains in the area of interest, which are hydraulicaly connected. This problem is similar to the double-porosity approach used in the models of transport in porous media, [6]. However, the double-porosity models use domains of the same dimension, with no potential flow in one of the domains and with the mass-exchange between the domains driven by diffusive processes. These three facts make a difference between our problem and the problem of transport in the double-porosity environment.

3. Approximation of the flow problem in each domain. We will show an approximation of the flow problem in each of the three domains without communication with the other ones in this section.

We have three domains Ω_i, i is an index denoting the dimension $i \in \{1, 2, 3\}$. Ω_1 is a set of mutually connected line segments placed in 3D space, Ω_2 is a set of mutually connected polygons placed in 3D space and Ω_3 is a simply connected three-dimensional domain. We can define a potential driven flow in each of these domains. The governing equations are the linear Darcy's law and continuity equation.

$$\mathbf{u}_i = -\mathbf{K}_i \cdot \nabla p_i \qquad \text{on} \quad \Omega_i, \tag{1}$$

$$\nabla \cdot \mathbf{u}_i = q_i \qquad \text{on} \quad \Omega_i, \tag{2}$$

where $\mathbf{u}_i$ is the velocity of the flow ($\mathbf{u}_2$ has to lie in the particular polygon, $\mathbf{u}_1$ has to have the direction of the particular line), p_i is the hydraulic pressure, $\mathbf{K}_i$ is the second-order tensor of hydraulic conductivity ($i \times i$ symetric, possitive definite matrix) and q_i is the function expessing the density of sources/sinks of the fluid. We prescribe three types of the boundary conditions on the $\partial\Omega_i$ — the Dirichlet's, the Neumann's and the Newton's:

$$p_i = p_{iD} \qquad \text{on} \quad \partial\Omega_{iD}, \tag{3}$$

$$\mathbf{u}_i \cdot \mathbf{n}_i = u_{iN} \qquad \text{on} \quad \partial\Omega_{iN}, \tag{4}$$

$$\mathbf{u}_i \cdot \mathbf{n}_i - \sigma(p_i - p_{iD}) = u_{iN} \qquad \text{on} \quad \partial\Omega_{iW}, \tag{5}$$

where p_{iD}, u_{iN} ans σ are given functions.

For the approximation of these three problems we use the Mixed-hybrid FEM with the lowest-order Raviart-Thomas elements on tetrahedras in Ω_3, triangles in Ω_2 and line segments in Ω_1. The rigorous formulation of the conituous problem and the derivation of the discretized problem can be found in [10] or [8] for the problem in Ω_3, in [11] for the problem in Ω_2 and in [12] for the problem in Ω_1.

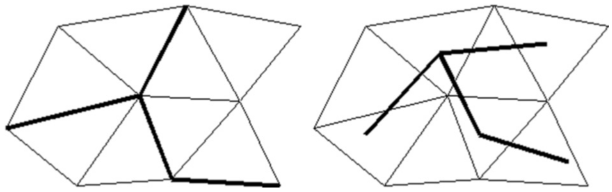

FIG. 1. *Example of the compatible and incompatible connection of the elements.*

The discretization leads to the system of linear equations in form:

$$
\begin{array}{llllll}
\mathbb{A}_i\mathbf{u}_i+ & \mathbb{B}_i\mathbf{p}_i+ & \mathbb{C}_i\lambda_i & = & \mathbf{r}_{i1} \\
\mathbb{B}_i^T\mathbf{u}_i & & & = & \mathbf{r}_{i2} \\
\mathbb{C}_i^T\mathbf{u}_i+ & & \mathbb{F}_i\lambda_i & = & \mathbf{r}_{i3}
\end{array} \tag{6}
$$

where λ_i are traces of the pressure on the sides of the mesh. We rewrite this system of equations in abbreviaed form

$$\mathbb{S}_i\mathbf{x}_i = \mathbf{r}_i, \tag{7}$$

where $\mathbf{x}_i = [\mathbf{u}_i, \mathbf{p}_i, \lambda_i]^T$, $\mathbf{r}_i = [\mathbf{r}_{i1}, \mathbf{r}_{i2}, \mathbf{r}_{i3}]^T$ and

$$\mathbb{S}_i = \begin{pmatrix} \mathbb{A}_i & \mathbb{B}_i & \mathbb{C}_i \\ \mathbb{B}_i^T & & \\ \mathbb{C}_i^T & & \mathbb{F}_i \end{pmatrix}.$$

4. Connection of the independent problems. We will show how to connect three independent problems presented in previous section and how to express the mass exchange between the domains Ω_1, Ω_2 and Ω_3. Due to properties of the mixed-hybrid formulation we can do that on the level of the discretized problem.

First, we join the three systems 7 into one large system

$$\mathbb{S}\mathbf{x} = \mathbf{r}, \tag{8}$$

where $\mathbf{x} = [\mathbf{x}_3, \mathbf{x}_2, \mathbf{x}_1]^T$, $\mathbf{r} = [\mathbf{r}_3, \mathbf{r}_2, \mathbf{r}_1]^T$ and

$$\mathbb{S} = \begin{pmatrix} \mathbb{S}_3 & & \\ & \mathbb{S}_2 & \\ & & \mathbb{S}_1 \end{pmatrix}$$

4.1. Compatible and incompatible connection of the elements. We allow two different kinds of connections of the elements with different dimensions, called *compatible* and *incompatible*. This fact makes our model unique from the most of other numerical models using the elements of different dimensions. These models (such FEFLOW) allow only compatible connections.

The difference between this two kinds of connection is shown in the figure 1.

The compatible connection requires the element of lower dimension placed exactly on the side or edge of the element of higher dimension. This connection seems to be natural way of connectig the elements in the FEM models, but it causes serious and almost unsolvable troubles at the stage of the mesh generation for problems with complex geometry of the domain. Unfortunately, the fractured rock massifs fall to this category of problems.

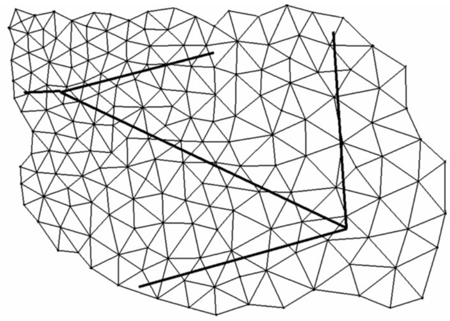

FIG. 2. *An unsuitable connection of the 1D and 2D elements — $h_1 \gg h_2$*

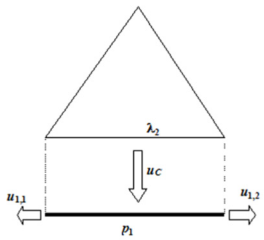

FIG. 3. *Fluxes and pressures for the compatible connection of 1D and 2D element. (The elements are drawn separated and shifted in the direction of the dotted line)*

This is the reason for allowing the other kind of the elements' connections — so called incompatible. In this case, there is no requirement on the spatial position of the communicating elements with different dimensions, the only requirement is on their sizes. We should use approximately the same discretization parameter h_i for all three meshes to avoid the situations shown in figure 2.

4.2. The mass exchange in the compatible connection between two elements. We will start the derivation of the equations for the mass exchange for the most simple case — compatible connection between two elements. The derivation will be shown on the case of 1D and 2D element, the case of the connection of 2D and 3D element is comletely analogous.[1]

The situation is drawn in figure 3.

First let us examine the original state. The marked side of the triangular element is considered as an external side of the 2D mesh. We assume the homegenous Neumann's boundary condition on this side

$$u_C = 0 \tag{9}$$

This equation can be found as one line of the block $\mathbb{C}_2^T$ of the matrix $\mathbb{S}_2$ and right-hand side $\mathbf{r}_{23}$. For the 1D element, there is a mass balance equation writen in the form

$$-u_{1,1} - u_{1,2} = 0, \tag{10}$$

[1] We do not allow direct compatible connection of 1D and 3D elements. If we need to incorporate the connection of this kind to our problem, we can do this indirectly by 2D element connecting both these elements or by setting the connection as incompatible.

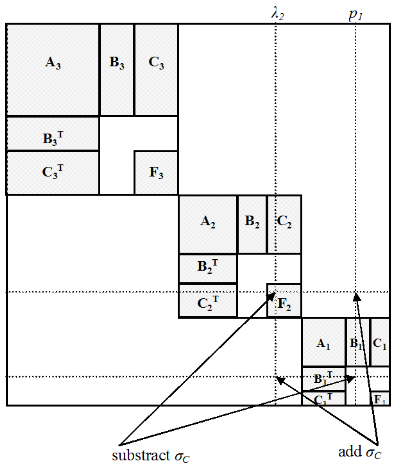

FIG. 4. *Setting up a compatible connection between 1D and 2D element in the matrix* $\mathbb{S}$.

which can be found in block $\mathbb{B}_1^T$ of the matrix $\mathbb{S}_1$ and the vector $\mathbf{r}_{12}$.

Now we express the exchange of the mass between the elements. We consider the flux u_C between 2D and 1D element is proportional to the pressure gradient between the elements

$$u_C = \sigma_C(\lambda_2 - p_1), \tag{11}$$

λ_2 is the pressure on the side of the 2D element, p_1 is the pressure in the centre of the 1D element and σ_C is the coefficient of proportionality. The mass balance equation for the 1D element can be written as

$$u_C - u_{1,1} - u_{1,2} = 0. \tag{12}$$

We can rewrite the equations 11 and 12 as

$$u_C - \sigma_C\lambda_2 + \sigma_C p_1 = 0, \tag{13}$$

$$\sigma_C\lambda_2 - u_{1,1} - u_{1,2} - \sigma_C p_1 = 0. \tag{14}$$

If we compare 9 with 13 and 10 with 14, we notice, that it is sufficient to add or substract cofficient σ_C to four elements of the matrix and we make the desired connection between the system $\mathbb{S}_1\mathbf{x}_1 = \mathbf{r}_1$ and $\mathbb{S}_2\mathbf{x}_2 = \mathbf{r}_2$. The changes in the matrix $\mathbb{S}$ are shown in figure 4.

4.3. Compatible connection of more than two elements. Now we will generalise the situation to the case of several elements of higher dimension connected with one element of lower dimension. Again, we will discuss this type of connection on 1D and 2D elements, the other type is analogous.

We have to emphasise that the sides of the triangular elements connected with the linear elements must be considered as external sides of the 2D mesh. There is no direct hydraulical connection between these triangular elements, such connection is possible only indirectly through the linear element. Therefore there is no pricipal difference between this case and the case discussed above. We take each of the triangular elements and we apply the procedure of setting up the connection between this

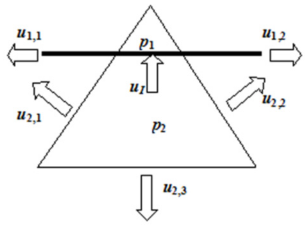

FIG. 5. *Fluxes and pressures for the incompatible connection of 1D and 2D element.*

particular triangular element and the linear element. Let there are n 2D elements with pressures λ_{2j}, $j \in \{1, \dots, n\}$ on sides connected with the 1D element with pressure p_1 and fluxes through its ends $u_{1,1}$ and $u_{1,2}$. The coefficents of the proportionality are σ_{Cj}. We got n equations of the form

$$u_{Cj} - \sigma_{Cj}\lambda_{2j} + \sigma_{Cj}p_1 = 0, \tag{15}$$

where u_{Cj} is the flux from the j-th triangular element to the linear one. For the linear element there is the mass balance equation

$$\sum_{j=1}^{n} \sigma_{Cj}\lambda_{2j} - u_{1,1} - u_{1,2} - p_1 \sum_{j=1}^{n} \sigma_{Cj} = 0. \tag{16}$$

We can incorporate the equations 15 and 16 to the system of linear equations 8 by the same process as it was shown above.

After adding values for all compatible connections (of both kinds — 1D with 2D and 2D with 3D elements) to the system 8, the matrix $\mathbb{S}$ changes its structure to this form:

$$\mathbb{S}_C = \begin{pmatrix}
\mathbb{A}_3 & \mathbb{B}_3 & \mathbb{C}_3 & & & & & & \\
\mathbb{B}_3^T & & & & & & & & \\
\mathbb{C}_3^T & & \mathbb{F}_{C3} & & \mathbb{E}_3^T & & & & \\
 & & & \mathbb{A}_2 & \mathbb{B}_2 & \mathbb{C}_2 & & & \\
 & & \mathbb{E}_3 & \mathbb{B}_2^T & \mathbb{D}_2 & & & & \\
 & & & \mathbb{C}_2^T & & \mathbb{F}_{C2} & & \mathbb{E}_2^T & \\
 & & & & & & \mathbb{A}_1 & \mathbb{B}_1 & \mathbb{C}_1 \\
 & & & & & \mathbb{E}_2 & \mathbb{B}_1^T & \mathbb{D}_1 & \\
 & & & & & & \mathbb{C}_1^T & & \mathbb{F}_{C1}
\end{pmatrix} \tag{17}$$

where $\mathbb{F}_{Ci}$ are modified blocks $\mathbb{F}_i$ and $\mathbb{E}_i$ are blocks created by the connecting of the elements.

4.4. The mass exchange in the case of the incompatible connection of the elements. In this section we will derive the equations describing the mass exchange between the elements of different dimmensions connected incompatibly. As in the previous section, we will show the derivation on the example of 1D and 2D elements, the procedure is the same for the other two cases (1D with 3D and 2D with 3D).

The situation is shown in the figure 5. The flux u_I between the elements is proportional to the pressure gradient as in previous case of the connection. We can express it like

$$u_I = \sigma_I(p_2 - p_1), \tag{18}$$

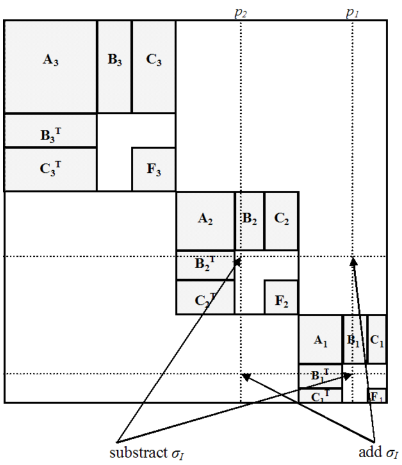

FIG. 6. *Setting up an incompatible connection between 1D and 2D element in the matrix* $\mathbb{S}_C$.

where p_2 is the pressure in the centre of the triangular element and p_1 is the pressure in the centre of the linear element. The coefficient σ_I has to reflect the size of the intersection of the elements and the distance of their centres. We write the mass balance equation for the triangular element

$$-u_{2,1} - u_{2,2} - u_{2,3} - u_I = 0, \tag{19}$$

where $u_{2,1}$, $u_{2,2}$, $u_{2,3}$ are fluxes through the sides of the triangle. For the linear element, the mass balance equation is

$$-u_{1,1} - u_{1,2} + u_I = 0, \tag{20}$$

where $u_{1,1}$, $u_{1,2}$ are fluxes through the ends of the linear element. If we substitute 18 to 19 and 20 we obtain:

$$-u_{2,1} - u_{2,2} - u_{2,3} - \sigma_I p_2 + \sigma_I p_1 = 0, \tag{21}$$

$$-u_{1,1} - u_{1,2} + \sigma_I p_2 - \sigma_I p_1 = 0. \tag{22}$$

As the original mass balance equations were

$$-u_{2,1} - u_{2,2} - u_{2,3} = 0,$$

$$-u_{1,1} - u_{1,2} = 0,$$

it can be seen that the incompatible connection of the elements can be realized by adding/substracting the value σ_I to elements of the matrix $\mathbb{S}_C$ shown in the figure 6.

This procedure can be repeated for each pair of the elements connected by the incomatible connection. The changes happen in the blocks $\mathbb{D}_i$ of the matrix 17, we call the changed block $\mathbb{D}_{Ii}$ and there are new blocs $\mathbb{G}_{ij}$

4.5. Some remarks concerning the connection of the particular models.

4.5.1. Rearrangement of the resulting matrix. The matrices produced by the MH-FEM models have some special properties. The most important of them is the positive deffinitnes of the block $\mathbb{A}$. The specialized solvers of linear equations uses this property of the matrix to make the process of solving more effective. Therefore it is wise to keep this property in our new model too. This goal is easy to achieve by a rearrangement of the state matrix, vector of solution and the vector of unknowns:

$$\mathbb{S}_{CI} = \begin{pmatrix} \mathbb{A}_3 & & & \mathbb{B}_3 & & & \mathbb{C}_3 & & \\ & \mathbb{A}_2 & & & \mathbb{B}_2 & & & \mathbb{C}_2 & \\ & & \mathbb{A}_1 & & & \mathbb{B}_1 & & & \mathbb{C}_1 \\ \mathbb{B}_3^T & & & \mathbb{D}_{I3} & \mathbb{G}_{32} & \mathbb{G}_{31} & & & \\ & \mathbb{B}_2^T & & \mathbb{G}_{32}^T & \mathbb{D}_{I2} & \mathbb{G}_{21} & \mathbb{E}_2 & & \\ & & \mathbb{B}_1^T & \mathbb{G}_{31}^T & \mathbb{G}_{21}^T & \mathbb{D}_{I1} & & \mathbb{E}_1 & \\ \mathbb{C}_3^T & & & & \mathbb{E}_2^T & & \mathbb{F}_{C3} & & \\ & \mathbb{C}_2^T & & & & \mathbb{E}_1^T & & \mathbb{F}_{C2} & \\ & & \mathbb{C}_1^T & & & & & & \mathbb{F}_{C1} \end{pmatrix} \tag{23}$$

As the right-hand side we use the vector

$$\mathbf{r}_{CI} = [\mathbf{r}_{31}, \mathbf{r}_{21}, \mathbf{r}_{11}, \mathbf{r}_{32}, \mathbf{r}_{22}, \mathbf{r}_{12}, \mathbf{r}_{33}, \mathbf{r}_{23}, \mathbf{r}_{13}]^T,$$

the vector of unknows has form

$$\mathbf{x}_{CI} = [\mathbf{u}_3, \mathbf{u}_2, \mathbf{u}_1, \mathbf{p}_3, \mathbf{p}_2, \mathbf{p}_1, \lambda_3, \lambda_2, \lambda_1]^T.$$

4.5.2. The domain Ω. In previous text we have consicered the domain Ω as

$$\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3,$$

providing that at least two of the three sets $\Omega_1 \cap \Omega_2$, $\Omega_1 \cap \Omega_3$, $\Omega_2 \cap \Omega_3$ are non-empty. This was a natural pressumption for the purposes of the derivation of the model. If we have model constructed by the above described procedure, we can weaken the requirement on Ω: It is sufficient to presume the Ω to be a simply connected set in the Eucidean space E_3.

$$\Omega = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3, \tag{24}$$

where Γ_1 is a set of line segments placed in the three-dimensional space, Γ_2 is a set of polygons placed in the three-dimensional space, Γ_3 is a set of three-dimensional domains.

4.5.3. The boundary conditions. The original requirement on boundary condition was the existence of the three non-empty parts of the boundary $\partial\Omega_{iD}$. We can weaken this requirement by the same way as we did for the domain Ω. Now it is sufficient to require only the existence of a non-empty part of the boundary $\partial\Omega_D$ of Ω, with prescribed the boundary condition of the Dirichlet's type.

5. Conclusion. We have introduced a one way how to set up a numerical model of the groundwater flow in the fractured rock environment. Our approach uses the mixed-hybrid FEM on three hydraulicaly connected domains.

We have implemented this approach in the programming language C. The resulting program is a subject of the testing at the time of writting of this paper. The first results of the testing shows, that the hydraulic communication and the mass exchange between elements of various dimensions works well and the behaviour of the groundwater calculated by our program has properties of behaviour of the groundwater in real fractured rock massifs.

There are some open problems and unanswered questions concerning this approach:

- Although the results of the tests seem to be quite possitive, we still know nothing about behaviour of our model in large, real-world hydrogeological problems.
- We have to define an algorithm for prescribing the values of the coefficients σ_C and σ_I. This algorithm will be based on our experiences gained by calculation of the real-world problems. These two coefficients will be good parameters for the calibraration of the models.
- Still there is no rigorous theoretical backround for the new model. We have proved the existence, uniqueness of the solution and the estimation of the error for all three models we used for the construction of the new one. These proofs for the new model are goals of the theoretical works in next months.

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